3.5 Poisson Statistics

This example has in mind a case where you collect several examples of a "rare" event (in the Poisson sense) – for example, in ten equal time slots you record $n_1, n_2 \ldots$ photons. Let us suppose that the true rate is ten photons. How well does the standard deviation of just the ten values compare with the true $\sqrt{10}$?
An easy way of examining this is by straight simulation; use your friendly random number generator to make sets Poisson-distributed numbers with a true (population) mean of 10. The graphs show the results as histograms for 1000 repetitions of this experiment, for ten and 100 time slots. Clearly, the result is much closer to the truth for more data (100 slots). Both of the distributions are quite skew, however. The *fractional* errors in the standard deviation are quite close to $1/\sqrt{10}$ and $1/\sqrt{100}$ – this is a good rule of thumb for estimating standard deviations.

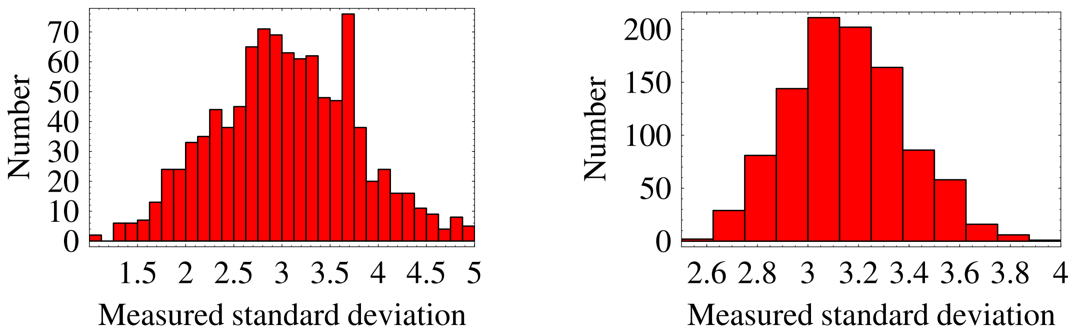

Figure 1: Distribution of measured standard deviation, true rate of 10 photons per slot, for 10 (left) and 100 (right) slots.

This suggests another experiment. Quite often we have just one slot and we estimate the standard deviation by $\sqrt{n_1}$. For a true rate of 10, what is the distribution of this estimate of the standard deviation? Figure 2 shows the result, again a simple simulation. This is a much more symmetrical distribution.

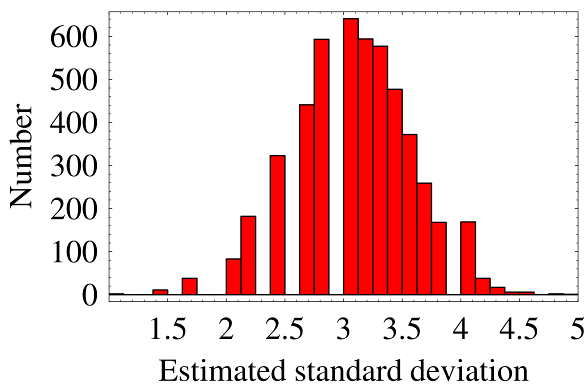

Figure 2: Distribution of estimated standard deviation, true rate of 10 photons per slot.